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Calculating the value of time

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1 Definition of the value of time: comparison with the wage rate

The marginal utility of time is usually defined as the Lagrange multiplier of the time constraint (total disposable time T equal to the sum of market work t_w and time used for domestic production and leisure t_d) : in such a model, households are supposed to maximize a direct utility depending on the time devoted to domestic production and leisure and on the quantities consumed of a Hicksian composite good containing all market goods whose relative prices are constant across the population. Under perfect substitution between market and domestic labor, the ratio of the marginal utilities of time and market expenditure equals both the opportunity cost divided by the price of the market good and the wage rate divided by the same price. The opportunity cost of time is thus supposed to be equal to the net wage rate. This equality is discussed in the first sub-section.

The model put forward in the following section is based on the estimation for each household of its opportunity cost of time (OCT) by means of the first order conditions for the maximization of a direct utility function depending on income and total disposable time in a domestic production framework. It thus differs from the methods used for instance in transport economics, which are based on the calculation of the Lagrange multiplier corresponding to a technological time constraint (defining for instance a minimum amount of time depending on the amount of transport expenditure), which necessitates specific identification constraints (see for instance Jara-Diaz et al, 2017a, b, c and DeSerpa, 1971, discussed in sections 2.5 and 2.6). In the model proposed in sections 2.7 and 2.8, a unique utility function is assumed for households, so that future research could analyze collective decisions and the allocation of goods and time within the family. As an example of an analysis in terms of collective decisions, Couprie and Ferrant (2012) test the separability between consumption and time use (separability is rejected for French statistics for couples but not for singles), and they analyze economies of scale, equivalence scales and welfare comparisons based on time use statistics. Intra-household distribution and welfare comparisons are also examined in recent interesting papers by Bargain and Donni (see for instance Bargain, Donni and Kwenta, 2014). Note that Bargain et al. conclude that 'imposing the restriction that parents behave as a single unit leads to similar results compared to the collective approach'.

Discussion on the difference between the wage rate and the opportunity cost of time

All direct surveys asking individuals to reveal their opportunity cost of time (OCT) give values which are much lower than their wage rate net of taxes, by at least 50%. For instance, the estimate derived by Aguiar and Hurst (2007b) looking at consumers' market behavior (time spent compared to expenditures) are around 1.5 per hour, a probable under-estimate of the OCT for other activities. Various reasons may explain a difference between the opportunity cost of time and the wage rate: whenever the two adults of a couple are not together employed in the labor market, the household OCT is linked to what the income opportunity of employment could be, either the minimum of the OCT of the

non-working adult (probably small if this adult has not entered the labor market: see the discussion by Gronau, 1973, pp. 637-641) and the wage rate of the employed partner, or the average of the estimated wage rate of the non-working adult and the wage rate of the employed partner (assuming the substitution of the incomes of both partners). The first case generally gives a smaller evaluation compared to the second case. A second reason is that all expenses due to working in the labor market (clothes, transportation costs, investment in human capital, etc.) must be deduced from the wage rate, as well as the marginal income tax (which rises with income). As a third reason, the productivity of the same individual for a similar task in the labor market or at home may be larger in the labor market, because of an adapted environment (computers, colleagues with complementary capabilities...) and human capital (all investments being paid by the employer). This implies a higher wage rate. Fourth, as noted by Folbre, 'most people lack the resources and the skills to successfully substitute home production to market purchased goods': in this case, people prefer to obtain a net wage (i.e. less taxes) by working in the market that is greater than their productivity at home, and then buy substitute goods or services in the market. Fifth, the wage rate and the OCT of individuals with low productivity in the labor market, should be equal to the official minimum wage rate. For other individuals with a higher productivity, the estimations show that the elasticity of the opportunity cost of time with respect to the wage rate is smaller than 1, so that on average the OCT would be smaller in the population than the average wage rate.

In France for instance, the estimated OCT is around 50% of the individual wage rate, and this ratio seems to diminish with the average level of income of a country (see a comparison between France, Canada, Ecuador and Guatemala in Canelas et al., 2017). On the contrary, the disutility linked to working in the market may be greater than the disutility of domestic work, because of the freedom implied by working at home and frequent joint production (e.g., discussions between partners while preparing meals, etc.). Also, constraints in the labor market impart a virtual cost corresponding to the Lagrange multiplier of each particular constraint. Such constraints occur, for instance, if actual working time is longer than the minimum imposed by the firm, or if working time cannot exceed a legal maximum. Finally, the opportunity cost of time may differ from the market wage, not only because of the disutility attached to market labor compared to home production, but also because of the possibility of joint production which characterizes the latter (see the discussion of this point in Gronau, 1977, and Apps and Rees, 2005, and a detailed analysis of this difference in sub-section 2.2).

2 Becker's theory of the allocation of time

In the Becker's original home production theory, the same opportunity cost of time applies for the time factor of the home production and on the labor market.

This value of time is thus supposed to be equal to the agent's wage rate. Becker considers a set of final goods (named also here activities) the quantities of which z_i, i=1 to I, enter the direct utility of the consumer $u(z_1, z_2, z_3..., z_I)$. In

order to simplify the analysis, Becker states that a separate activity i produces the final good i in quantity z_i using a unique market good[1] in quantity x_i and time t_i together with all other (socio-economic) resources or characteristics of the household in a vector of covariates G: $z_i = f(x_i, t_i; G)$. The consumer's program writes:

such that $z_i = f(x_i,t_i; G)$ under the constraints on resources

$$\sum_i p_i x_i = y \text{ and } \sum_i t_i + t_w = T \tag{1}$$

with $y = wt_w + y_0$ the monetary income which sums the labor income with all other incomes y_0 (financial, social transfers, monetary gifts...), t_w the labor time on the market and T the total disposable time for one period[2].

Becker makes a further assumption about the *complementarity* of the factors of domestic production (market goods and time): θ_i is the time use necessary to produce one unit of the activity i, a Leontief technology implies that the amount of time t_i which is necessary to produce a quantity z_i of the activity i is:

$$t_i = \theta_i z_i \tag{2}$$

A similar assumption concerning the market good supposes that the quantity of the market good is also proportional to the level of the activity:

$$x_i = \xi_i z_i \tag{3}$$

so that finally: $\frac{t_i}{x_i} = \frac{\theta_i}{\xi_i}$: the ratio of the unknown coefficients of production θ_i and ξ_i can be therefore recovered by the ratio of the quantities of the two factors of production necessary to produce a quantity z_i of the activity.

3 Empirical valuations of the value of time

These empirical studies confirm that the economic agents value positively an increase of their disposable time and compare the evaluations of time devoted to special activities, such as market work, leisure and domestic production.

Transportation studies

The trade-off between time and monetary resources was empirically examined in various studies before the theoretical analysis by Becker in 1965, especially in transportation studies. For instance, Beesley (1965) studies the value of time spent traveling in various studies based on English panels on road pricing in order to analyze the cost of road congestion. Time saving accounts for 64% to 78% (according to the income status of the household) of the measured gross

[1] In case multiple market goods are used in activity i, a generalization to a bundle of market goods used to produce the activity can be performed by defining an aggregate commodity of the market goods corresponding to this activity. The monetary price π_i will be defined in this case as a price index for this aggregate, coherent with the monetary budget constraint.

[2] In the consumer program below, the domestic production functions will be written in terms of monetary expenditures ($m_i = p_i x_i$) instead of the quantity consumed since generally the latter is not observed in the Family Budget surveys. Prices are thus supposed to be the same for all households in the survey.

benefit at the first year of a public operation reducing congestion. In that estimation, saving in working time is valued at 7 shillings, compared to 5 shilling for saving non-working time. His study concludes: first, there exist some trading behavior between transport time and monetary expenditures for at least one third of population; second, women seem to be more non-traders than men(but this observation is based on a limited number of observations); third, the income elasticity of the value of time is estimated at 0.6; fourth, the ratio of the value of time to the average wage rate varies from 31% to 50%[3]; finally, the value of time is a little higher for transportation by one's own car than by public transport (indicating that some direct utility, independent from the result of the travel, may proceed from transport by the agent's car).

Valuation of domestic production

All studies published after 1965, even the pure empirical ones, make some direct or indirect reference to Becker's seminal work. For instance, an interesting study by Murphy (1978) evaluates the domestic production in the U.S. using either the market value of domestic work (market wage rate assigned to household task such as food preparation, clothing maintenance, house upkeep or family care[4]) or an opportunity cost based on the household's marginal wage rate net of taxes.These two methods give similar results (36.8% of the GDP for the first compared to 37.7% for the second), these proportions remaining stable over the period (1960-1970). The substitution between home work (implying some disutility or positive utility of the activity [5]), leisure and market work depends on the relationship between the time values of the components of the allocation of disposable time: the value of disposable time for leisure ω , (i.e. of leisure, leisure being an activity defined by the absence of a direct utility), the opportunity cost of timeω[6], equalized with the sum of the household's wage rate net of taxes W and marginal direct utility of work (expressed in money terms, i.e. divided by the marginal utility of income: $U_w = \frac{\frac{\partial U}{\partial t_w}}{\frac{\partial U}{\partial y}}$), or with the sum of the market cost of home activity W_d (expressed in money terms: $W_d = \frac{\frac{\partial U_1}{\partial t_d}}{\frac{\partial U}{\partial y}}$) and the marginal utility of domestic work (in money terms: $U_d = \frac{\frac{\partial U_2}{\partial t_d}}{\frac{\partial U}{\partial y}}$):

[3]Note that the value of time equals the wage rate when observed during (or near) working time (p. 183). However, it could be smaller than the wage rate if market work is associated with a negative direct satisfaction.

[4]Note that this value differs from a replacement cost indicated by the annual earnings of full-time domestic servants, which is probably greater than the cost of the corresponding market service.

[5]*Direct (intrinsic)* utility U_2 of some domestic work (or work time) indicates an independent component of satisfaction U_1 derived from the utility derived from the result of the activity. Market work generates for instance an income which gives rise to the satisfaction of household's wants through market consumption and saving, but there exist also some satisfaction or disatisfaction caused by the fact to work outside, to be constrained by legal obligations or bad conditions at work, or simply to be obliged to work at hours decided by the direction. Total utility thus writes: $U = U_1 + U_2$. The consequences of this at decomposition will be discussed in section 2.5.

[6]Note that the opportunity cost is supposed to remain the same for all home activities, while the market value almost doubles between Family care and Food preparation. That difference between the valuation of different home activities will be discussed later.

$$\omega = W + U_w = W_d + U_d \tag{4}$$

As the marginal utility of market work is supposed to be negative and the marginal utility of home activities positive or smaller in absolute value, $U_w < U_d \Rightarrow W_d < W$ and $\omega > W_d$ if $U_d < 0$, so that the market cost W_d places a lower bound to the valuation of domestic production (compared to valuations by the value of disposable time ω, the opportunity cost W and the market cost W_d. The U.S. statistics (1960 and 1970) used for the calculations confirms this difference but it seems to be quite small (3 to 8%).

Gronau's valuation of housewives' time (1973)

In one of the first article he published on the subject, Gronau modelizes the intrafamily allocation of time between market work and domestic production by two spouses (i=1,2) considering that the utility of the family depends on the amount of market goods M and home goods H and on the husband's and wife's leisure times L_1 and L_2: $U(M, H, L_1, L_2)$. Home goods are produced with a combination of market good (in quantity x) and time T_h by either husband or wife: $H_i(X_i, T_{H_i}) = min(\frac{X_i}{\gamma_i}, \frac{T_{H_i}}{\delta_i})$ with complementary factors (i.e. with a fixed proportion technology): $X_i = \gamma_i H_i, T_{H_i} = \delta_i H_i$. The marginal cost of producing one unit of home goods ($H_i = 1$) by the individual i is: $\pi_i = \gamma_i p_x + \delta_i \omega_i$ with ω_i the value of her time. Work is supposed to carry no utility. Two cases are examined in the allocation of time of the family: either both members participate in the labor force, and in this case home goods are exclusively produced by the wife if her wage rate is smaller than her husband's, or only the husband participates in the labor force because the value of her time (being positively linked to the household's income) exceeds her potential wage: in this case, the domestic production will be made either only by the wife or by both members.

The first order conditions of the maximization of the utility of the household under monetary and time constraints are examined in the two cases of both members participating in the labor force and only the husband participates. In the first case, the value of time of each member equals one's wage rate W_i, thus defining the marginal cost of producing home goods by spouse i as depending on her marginal productivity (γ_i and δ_i) and her wage rate:

$$\pi_i = \gamma_i p_x + \delta_i W_i \tag{5}$$

Supposing that the husband's wage exceeds his wife's one, all domestic production will be made by the wife while the husband divides his time between market labor and leisure. This allows to define the full price of domestic productions and the household's full income as depending on the husband's wage rate, yielding a unique Lagrange multiplier as a measure of the marginal utility of full income. In the second case where only the husband participates in the labor market, an increase in the household's income increases the value of the wife's time which may exceed her potential wage, thus deciding her to leave the labor market and dividing her time exclusively between work at home and leisure. As a consequence, the value placed on the wife's time can be sufficiently large to make husband-produced and wife-produced home goods equally costly, so that the husband may be forced to participate to the production of home

goods.

The probability of the first case (working women, characterized by a value of time greater than their wage rate: $Prob[\omega > W]$) being supposed to follow a normal law, a linear relation follows between the value of time and the wage rate (see Gronau, pages 643-645):

$$\bar{W} = \mu_\omega + Z\sigma_\omega \tag{6}$$

with μ_ω and σ_ω the mean and standard error of the value of time, $Z = \frac{\mu_w - \mu_\omega}{\sigma_\omega}$ under the assumption: (H1) $\sigma_W = 0$ and σ_ω is insensitive to income.

An alternative assumption: (H2) $\sigma_\omega = 0$ and σ_W is independent of age and education (defining the sub-populations on which the estimation will be processed) gives rise to the linear relation:

$$\bar{W} = \mu_\omega + Z^*\sigma_\omega \tag{7}$$

with $Z^* = \frac{\mu_w - \mu_\omega}{\sigma_w} + x$ where x is the average $\frac{w - \mu_w}{\sigma_w}$ for $x > -Z^*$.

The first assumption ($\sigma_W = 0$) supposes that working women are those who have the lowest price (being the least productive at home) while by the second assumption ($\sigma_\omega = 0$), women who work are the most productive in the market. According to the first assumption, housewives' average value of time is expected to exceed the average wage rate, while the reverse holds for the second assumption. The two estimates yield in fact the extreme limits for the mean value of time.

The estimation of these linear relations on a set of 768 sub-populations of wives (defined by age and education) allows to calculate probabilities of the participation of wives to the labor market and thus the mean and standard error of the value of time. For the preferred second assumption ($\sigma_\omega = 0$), the average value of time (Table 3 page 648) is estimating as 80% of the wage rate for the whites and 74% for the nonwhites, with a positive correlation with the household's income (income elasticity is 0.34 for the whites and 0.14 for the nonwhites - but 0.48 for nonwhites with one young child) [7]. An important conclusion is that the common practice to equate the value of time of housewifes with that of working women (i.e. to their expected wage rate) may involve an error close to 20% for white housewives and 50% for nonwhites[8].

4 Generalizations and complements to Becker's model

Oort's evaluation of traveling time (1969)

Oort's model is designed as a maximization under time and money constraints as a means to evaluate the social benefit derived from a reduction of traveling

[7] Note that household's income explains no more than one-half of the difference between whites and nonwhites.

[8] Note that this analysis is made under the assumption that work in the market and home production do not involve any direct (intrinsic) utility.

time[9]. Working time being supposed fixed (choosen in a first stage), a reduction of traveling time is equivalent to an increase in the effective time available to the agent for either work or leisure.

The individual utility depends on the amount of working time t_w and leisure t_d[10] and money income $Y = Wt_w$:

$$U = U(t_w, t_d, Y) \tag{8}$$

where the amount of working time produces directly some (dis)utility U_2 which adds to the indirect utility derived from labor income[11].

The utility is maximized under the monetary: $Y = wt_w$ and time constraints: $t_w + t_d = T$, which gives the Lagrange multipliers:

$$\frac{\partial U}{\partial t_w} = -(\lambda + \mu W) \tag{9}$$

$$\frac{\partial U}{\partial t_d} = -\lambda \tag{10}$$

$$\frac{\partial U}{\partial Y} = -\mu \tag{11}$$

which implies that the marginal utility of domestic production (leisure in Oort's model) equals the marginal utility of the the money earned by spending the time in work, plus the marginal (dis)utility of labor (indicating the marginal net benefit of labor) :

$$\frac{\partial U}{\partial T} = -\frac{\partial U}{\partial y_d} = \frac{\partial U}{\partial t_w} + \frac{\partial U}{\partial E_w} \tag{12}$$

where $\frac{\partial U}{\partial E_w}$ stands for the direct intrinsic (dis)utility of working (E_w depending on the labor supply t_w and the characteristics of the job. Domestic production can involve also some direct (dis)utility $\frac{\partial U_2}{\partial E_d}$ (which could be positive for some domestic activities such as children caring) which gives rise to the total effect of a change in domestic time[12]:

$$-\frac{dU}{dt_d} = \frac{\partial U}{\partial t_w} + \frac{\partial U}{\partial E_w} - \frac{\partial U}{\partial E_d} \tag{13}$$

This implies that the marginal net benefit of labor represents a lower limit to the total net (dis)utility of reducing domestic time (travel time in Oort's analysis[13]). Therefore, the value of domestic time writes:

[9]Social cost benefit studies around the 60's conclude that time savings accounted for well over one half the total benefits.

[10]In Oort's model, domestic time is the sum of travel time and pure leisure time.

[11]$U = U_1 + U_2$, U_1 representing the satisfaction derived from the activity (which adds to the utility of the domestic production made by using time t_d.

[12]see the straighforward calculus in Oort, note 7 page 283 and its interpretation:"'At the margin, the individual's evaluation of traveling time equals the utility he would obtain if he were to spend that time in the form of additional work (what have been called the net benefit of labor) plus the benefit consisting in the reduction of the time during which he suffers the disconforts of travel"'

[13]Leisure being supposed to produce no intrinsic satisfaction

$$\omega_{tr} = \frac{-\frac{dU}{dt_d}}{-\frac{dU}{dY}} \tag{14}$$

which is smaller than the marginal rate of wage if the (negative) marginal direct utility of market work ($\frac{\partial U}{\partial E_w} < 0$) is smaller, in absolute value, than the direct (dis)utility of domestic time ($\frac{\partial U}{\partial E_d}$). Note that the level of disconfort of travel depends on household's characteristics[14], and the type of domestic activity, which implies different values of domestic times even if the wage rate and the disconforts of market labor do not depend on the type of domestic activity.

Oort's analysis explains why the net benefit of market labor can differ between households (characterized by the same wage rate) because of different disutilities of market labor. The second important conclusion is that the money compensation of the disconforts attached to each domestic activity can explain the variance of the values of time used for the different domestic activities by the same household[15].

DeSerpa's general model (1971): technological constraints on time

DeSerpa generalizes Becker's model following Oort's analysis by adding the time uses for different domestic activities t_i as arguments of the utility function:

$$U = U(x_1, ..., x_n, t_1, ..., t_n) \tag{15}$$

and adding technological constraints on the minimum time by activity depending on the associated consumption of the market good for commodity (i.e. domestic activity) i:

$$t_i \geq a_i x_i \tag{16}$$

This inequality corresponds to a *time consumption (technological) constraint* while the usual budget time constraint writes:$T = \sum_i t_i$.

That generalization gives rise to the following Lagrange function:

$$L = U(x_1, ..., x_n, t_1, ..., t_n) + \lambda(Y - \sum_i p_i x_i) + \mu(T - \sum_i t_i) + \sum_i \kappa_i(t_i - a_i x_i) \tag{17}$$

Note that in this equation time uses appear as, first associated with market goods in the domestic production, but also affording direct (dis)utility corresponding to the (dis)conforts associated with the domestic activity), contrary to the usual case where this direct (dis)utility is not taken into account and time uses are only factors of the domestic production functions together with market goods. The utility can be written :

[14] increasing for instance with the wage rate since the individual can earn the money to pay the disconforts easier if the compensation in market labor and the disconforts attached to it are smaller

[15] Note that the value of time for a given activity can change also according to the member of the family who makes that domestic production.

$$U = U_1(x,t) + U_2(t_1,...,t_n) \tag{18}$$

with a utility of time use U_1 and a direct (dis)utility of the activity U_2.

The first order conditions writes with a marginal utility of time use U_{1i} and a direct (dis)utility of the activity U_{2i}:

$$U_{1i} = \lambda p_i + \kappa_i a_i \tag{19}$$

$$U_{2i} = \mu - \kappa_i \tag{20}$$

$$\kappa_i(T - a_i x_i) = 0 \tag{21}$$

corresponding, either to a binding constraint: $t_i = a_i x_i$ (in that case, $\kappa_i > 0$) or to $\kappa_i = 0$ if the actual time use t_i^* is greater than the lower limit $\bar{t}_i = a_i x_i^*$ with x_i^*, t_i^* the optimal choice of the individual.

The value of time is the ratio of the complete marginal utility $U_i = U_{1i} + U_{2i}$ and of the marginal utility of income indicated by the Lagrange multiplier λ:

$$\omega_i = \frac{U_{1i} + U_{2i}}{U_Y} = \frac{\mu - \kappa_i}{\lambda} \tag{22}$$

This value of time is smaller (for activities characterized by a binding time constraint) than the usual $\frac{\mu}{\lambda}$ corresponding to Becker's model (with no (dis)utility of the domestic activity) since the multipliers κ_i corresponding to the technological inequalities are always positive. $\frac{\kappa_i}{\lambda}$ can be interpreted as the *value of saving time from activity i* (characterized by a binding constraint) since it equals the value of time as a resource ($\frac{\mu}{\lambda}$ which is the value of time in alternative uses) minus the marginal utility of time used (divided by the marginal utility of income λ) for that particular activity.

Section 2 of DeSerpa's article examines the quantities demanded of each commodity which can be derived from the first order conditions and concludes that the value of saving time from activity i can be derived from a log-linear demand function:

$$log(x_i) = \alpha_i + \beta_i log(p_i) + \gamma_i log(a_i) + \delta_i log(Y) + \varepsilon_i \tag{23}$$

in terms of the parameter of this demand:

$$\frac{\kappa_i}{\lambda} = \frac{\frac{\gamma_i}{a_i} - \left[\frac{\partial(\frac{\kappa_i}{\lambda})}{\partial p_i} + x_i \frac{\partial(\frac{\kappa_i}{\lambda})}{\partial Y}\right]}{\frac{\beta_i}{p_i}} \tag{24}$$

which could be estimated according to DeSerpa by an iterative method on the partial derivatives of $\frac{\kappa_i}{\lambda}$ (supposed in a first round to be equal to zero).

This analysis makes an important distinction between the value of time as a resource for all types of activities (or of leisure time, leisure being considered to be not submitted to a technological consumption constraint) and the value

of time saving for a particular activity. The first value correspond to the opportunity cost of leisure, the second to the supplementary value arising from the the intrinsic utility and the inequality on time use derived from a technological constraint on domestic production with time. The existence of a technological time constraint is thus a second reason why the opportunity cost of time for a particular activity may differ from the net wage rate corresponding to the value of time on the labor market.

Interpretation in terms of discrete choice models (Truong and Hensher, 1985)

Truong and Hensher show that the DeSerpa model can been derived from a discrete choice model applied to the choice between different modes of transport (such as public and private urban transport or a qualitative change to a more confortable mode). The individual is supposed choosing a transport mode i out of the set of I discrete alternatives corresponding to the maximum of an indirect utility function depending on the prices π_{ij} of the different alternatives (j indicating the attributes of alternatives i, such as their monetary cost and travel time).

$$V = V(\pi_{ij}, i = 1ton, j = 1tom) \tag{25}$$

This indirect utility corresponds to the maximum of a direct utility U depending on the quantities consumed of a market aggregate good x and leisure time t_l after deducing from income and disposable time travel cost C_i and travel time t_i. Considering the usual monetary and time constraints, they obtain the Lagrange multipliers $\lambda = \frac{\partial U}{\partial x}$ and $\mu = \frac{\partial U}{\partial t_l}$ and the shadow price of time for the maximum utility as:

$$\pi_i = \frac{dC_i}{dt_i} = \frac{dx}{dt_l} = \frac{\frac{\partial U}{\partial t_l}}{\frac{\partial U}{\partial x}} = \frac{\mu}{\lambda} \tag{26}$$

Adding the DeSerpa constraint $t_i \geq a_i x_i$ gives rise to the fist-order condition similar to DeSerpa's analysis:

$$\frac{\partial u}{\partial t_i} = \mu - \kappa_i \tag{27}$$

with κ_i the marginal utility of releasing the technological constraint of travel time. It corresponds to the marginal utility cost to spend one more unit of time in order to increase an optimum amount t_i^* up to $a_i x_i = t_i^* + 1$ or, in Truong-Hensher terms, to "'the value one can gain if technological changes enable one to move closer to its optimum t_i^*"' in the Becker's model (without technological constraints). This value of travel time differs from the value of leisure time (leisure being characterized as an activity with no technological constraint on time) and writes:

$$\frac{\frac{\partial U}{\partial t_i}}{\frac{\partial U}{\partial x_i}} = \frac{\mu - \kappa_i}{\lambda} \tag{28}$$

This optimization results in an indirect utility function depending on the values of the attributes of the preferred mode, represented by the Lagrange

multipliers λ, μ, κ_i[16], from which one can derive a probabilistic discrete-choice function by adding a random component which accounts for probabilistic variations in tastes:

$$V_i = V(\lambda, \mu, \kappa_i) + \varepsilon_i \tag{29}$$

This residual being assumed to follow an extreme-value distribution, the probability of choice is distributed as a logistic function:

$$P_i = \exp \frac{\theta V_i}{\sum_i \exp \theta V_i} \tag{30}$$

For the application, Truong-Hensher use linear specifications of the indirect utilities (with $m_i = p_i x_i$ the expenditure for market goods):

Becker's model

$$V_i = \alpha_i - \lambda m_i - \mu t_i \tag{31}$$

DeSerpa's model

$$V_i = \alpha_i - \lambda m_i - (\mu - \kappa_i) t_i - \sum_{j \neq i} \kappa_j t_j \tag{32}$$

and estimate a Multinomial Logit Choice Model in order to estimate the Lagrange multipliers which results in a shadow price of time $\frac{\lambda}{\mu} = 12.09$ Australian dollars (1982) per hour[17] for the Becker's model.

Dividing travel time into three alternative modes (car, bus, train) results in the estimation of DeSerpa's model and of the shadow prices $\frac{(\mu - \kappa_i)}{\lambda}$ [18] which gives rise, by a substraction with the estimation of $\frac{\mu}{\lambda}$ obtained in the Becker's model, the estimates of $\frac{\kappa_i}{\lambda}$ for car travel (-4.74 Australian dollars (1982) per hour), Bus travel (+3.24) and Train travel (+3.18), with an overall weighted average equal to 2.09 dollars. This is the reason why there exist a difference between the average estimation of the shadow prices of time estimated for each activity and the value of time corresponding to leisure (supposed to free of any technological time constraint): the empirical application on travel time in Truong-Hensher's paper shows that a Becker model overestimates the value of travel time (12.09 dollars) by 17%.

Table 2.1: Estimates of the value of time

Value of time	Becker's model	DeSerpa's model*	DeSerpa's model**
RVT	12.090		
AVTSA	-	9.996	9.732
VTSA Car	-	16.860	16.326
VTSA Bus	-	8.880	6.498
VTSA Train	-	8.940	7.110
W	-	-	9.258

[16]Note that in the Becker's model the indirect utility depends on only the two first multipliers, excluding κ_i. It is thus possible to compare the estimations of the value of time of the Becker's and DeSerpa's models.

[17]One Australian dollar is equivalent to 1.1 U.S. dollar.

[18]Which does not include, in DeSerpa model, the marginal utility of the activity

Note: Tables 1 to 3 from Truong-Hensher (1985). Value of one hour in 1982 Australian dollar (= 1.1 U.S. dollar). *Becker's model*: indirect utility regressed on monetary expenditure and time use for travel; *DeSerpa's model* (Table 2)*: travel time disaggregated into car use, bus and train; *DeSerpa's model** (Table 3)*: travel time disaggregated into total out-of-vehicule time and disaggregated invehicule time uses for car, bus and train. RVT: resource value of time=$\frac{\mu}{\lambda}$; AVTSA: average value of time spent in all activity=$\frac{(\mu-\bar{\kappa_i})}{\lambda}$; VTSA: value of time spent in activity=$\frac{(\mu-\kappa_i)}{\lambda}$; W: average gross wage rate of sample.

Interpretation: a possible link with the degree of substitution between domestic production factors

The multipliers of the time technological constraints κ_i must always be positive, so that the negativity of the car travel parameter estimated by Truong-Hensher is caused by a negative marginal utility V_i which is included in the estimate of κ_i in the DeSerpa specification[19]. Separating these two components V_i and κ_i needs some assumption, for instance the equality of the marginal utilities in the comparison of two activities.

Becker's estimate of the resource value of time (i.e. corresponding to non binding activities, such as leisure) overestimates the value of travel time (average value of transferring travel time to a leisure activity) by 2.09 in the first estimation of DeSerpa's model, and by 2.36 in the second. Therefore, taking into account the technological constraints to time travels brings together the value of time and the wage rate. The value of time spent in travel differs between car use and the two other travel modes: transferring one hour spent in car to a bus travel decreases the traveler budget by 7.98 dollars, while the value of time is similar for bus and train travels. If the Lagrange multipliers of the time constraints are the same for all travel modes (i.e. if they have similar technological time constraints, which is not very probable since the ratio of monetary over time expenditures differs much between these modes), the high level of the value of time for car travel indicates that traveling by one's own car is much more satisfying than public transport (but also probably more costly). It is also possible that the technological constraint is more binding (i.e. the optimum is further away to the minimum time) for the first mode, which would give rise to a larger Lagrange multiplier. The fact that the estimated sum of the value due to the technological constraint ($\frac{(\mu-\kappa_i)}{\lambda}$) and of the direct marginal utility of the activity are similar for bus and train travel seems to indicate that these two components are the same for the two last travel modes.

Test of the Becker's specification

Becker's model is a special case of DeSerpa's, constrained by the nullity of all Lagrange multipliers κ_i (i.e. with no technological constraint over time uses). DeSerpa model writes, in Truong-Hensher's article:

$$V_i = \alpha_i + \beta_i x_i + \gamma_1 t_1 + \gamma_2 t_2 + \gamma_3 t_3 + \delta Z \tag{33}$$

[19]Note that this component could also be negative for the other items, but with a value which is not sufficient to compensate the positive time constraint multiplier.

where V_i is the indirect utility for alternative i, the indices indicating the three travel mode (car, bus, train) and Z a set of socio-economic covariates. Becker's model corresponds to the case where $\gamma_1 = \gamma_2 = \gamma_3$ or $\kappa_i = 0, i = 1, 2, 3$.

The comparison of the log-likehoods of Becker's and DeSerpa's models (Truong-Hensher, 1987) indicates that they differ statistically at a confidence level of 1%, which means that the Becker's hypothesis of an equal valuation of travel time among different modes is rejected. As a consequence, the overall value of travel time differs from the resource value of time corresponding to leisure activities.

The *valuation of time spent* ω_I in the semi-aggregate activity I (comprising sub-activities i) writes:

$$\omega_I = \sum_{i \in I} \frac{t_i}{t_I} (\frac{\mu - \kappa_i}{\lambda}) = \frac{\mu}{\lambda} - \frac{t_{I_c}}{t_I} \sum_{i \in I^c} \frac{t_i}{t_{I_c}} \kappa_i \tag{34}$$

which is the weighted sum (with weights $\frac{t_{I_c}}{t_I}$ and $(1 - \frac{t_{I_c}}{t_I})$) of the *valuation of time spent in the non binding sub-group* I_{nc}: $\omega_{I_{nc}} = \frac{\mu}{\lambda}$, and the *time valuation in the binding sub-group* I_c: $\frac{\mu}{\lambda} - \frac{t_{I_c}}{t_I} \sum_{i \in I_c} \frac{t_i}{t_{I_c}} \kappa_i$. As this second component is positive (being the average of positive individual ω_i in sub-group I_c), the value of time ω_I is also positive but smaller than the *resource value of time* $\frac{\mu}{\lambda} - \kappa_i$ corresponding to non-binding (leisure-type) activities.

5 Other econometric methods for the evaluation of the implicite opportunity cost of time

5.1 Indirect method based on the comparison of money and time measures of the ordinary surplus (Bockstael et al., 1987)

Another original method for the estimation of the opportunity cost of time consists in estimating the welfare losses associated with a hypothetical elimination of some resource (such as for instance recreational fishing). Comparing the money and time measures of the ordinary surplus or of the compensating or equivalent variations necessary to compensate individuals for loss of the resource, allows to calculate the implicite value of time which equalizes monetary and time valuations (see some detail on the calculation of Bockstael et al. (1987) in Appendix). This article evaluates the demand for recreational fishing conditional on the recreationist's labor market situation, either constrained on the number of working hours (implying a corner solution) or free to choose it (interior solution). In the first case, utility maximization under two constraints leads to a demand function with travel monetary costs and travel time as independent variables (and a value of time corresponding to the Lagrange multiplier of the time constraint), while the latter implies that time is valued at the wage rate and leads to the combination of monetary and time costs into a full cost variable. A linear specification of these demand functions allows to compute (for the two possible situations on the labor market) the usual consumer surplus associated to the existence of the recreational activity, and the compensating and equivalent variations. The ratio of these welfare measures in monetary and time gives rise to an implicite valuation of the value of time corresponding to

the estimation of the demand functions.

The three welfare indexes give rise to close valuations of the value of time: for instance 60 U.S. dollars (1983) per hour for individuals with fixed work hours (corner solution to the maximization of utility) and 17 for those with flexible work hours (interior solution), thus implying a large virtual cost of the labor constraint (43 dollars per hour). This virtual price will be discusses later in chapter IV.

5.2 Evaluation by a system of demand including goods and activities (Jara-Diaz et al., 1998-2008)

Jara-Diaz and his co-authors published recently a series of papers which derive a model of consumer behavior including market goods and activities (measured by their time uses). This model is based on a Cobb-Douglas specification of the direct utility of labor time on the market t_w, uses of time t_i for activities i and consumption of market goods j in quantities x_j :

$$U = \alpha t_w^{\theta_w} \prod_i t_i^{\theta_i} \prod_j x_j^{\phi_j} \tag{35}$$

where α is a constant and θ_w, θ_i and ϕ_j parameters of time uses and consumption of goods. This utility is maximized under an income constraint (involving a Lagrange multiplier λ), a constraint on total time resource (with Lagrange multiplier μ) and technological constraints on time uses and consumptions (with Lagrange multiplier κ_i and eta_j).

The first order conditions for goods, domestic time and market labor imply a linear dependency of market labor and freely chosen domestic time and goods expenditures over commited expenses E_c and commited time T_c, the wage rate W, market labor and total disposable time T, with a large number of parameters to estimate. Some simplifying assumptions give rise to the following equation for the market labor supply:

$$t_w = (T - t - c)\left[(\beta + \alpha e_c) + \sqrt{(\beta + \alpha e_c)^2 - e_c(2\alpha + 2\beta - 1)}\right] \tag{36}$$

with $e_c = \frac{E_c}{W(T-t_c)}$ the ratio of the minimum monetary expenditure (commited expenses) over the maximum possible income[20]. The marginal utilities of income λ and time μ given by the first order conditions yield the estimate of the value of leisure time ω and of market work time VTW:

$$\omega = \frac{\mu}{\lambda} = \frac{1 - 2\beta}{1 - 2\alpha} \frac{Wt_w - E_c}{T - t_w - t_c} \tag{37}$$

$$VTW = \frac{\frac{\partial U}{\partial t_w}}{\lambda} = \frac{2(\alpha + \beta) - 1}{1 - 2\alpha} \frac{Wt_w - E_c}{t_w} \tag{38}$$

[20]The household's labor supply varies from a fraction $2\beta(T - t_c)$ of available time to this available time $(T - t_c)$.

The model is estimated in Germany, Swiss and Chile over limited surveys of individual weeks (with small samples size from 124 to 325 observations) and gives estimates of the value of time indicated in Table 2.2:

Table 2.2:Average values of time (Chile, Germany, Swiss)

Value	Santiago			Karlsruhe			Thurgau		
	Value	t-stat	% wage	Value	t-stat	% wage	Value	t-stat	% wage
Leisure	2.9	24.1	65.9	12.7	4.0	119.8	26.7	5.6	87.8
Work	-1.5	-15.3	34.1	2.1	0.7	19.8	-3.7	-0.8	12.2
W	4.4	-	100	10.6	-	100	30.4	-	100

Note: Table 3 from Jara-Diaz et al. (2008). Value of one hour in 1999-2003 (1982 U.S. dollar). *Leisure*: RVT= resource value of time=Value of leisure time: $\omega = \frac{\mu}{\lambda}$. *Work*: VTW Value of work time: $\frac{\frac{\partial U}{\partial t_w}}{\lambda}$. W: average wage rate.

The resource value of time is thus positive, as expected for the ratio of two positive marginal utilities. It is (except in the case of Germany, which estimates are less significant), smaller than the wage rate by one quarter in average for Chile and Swiss, which indicates a disutility of market labor time as expected. Another previous estimation by Jara-Diaz and Guevara (2003) on the same chilean survey (without the technological constraint on monetary expenditures) gives quite different results, which shows that estimating all parameters on the labor supply function (with some simplification hypothesis in order to obtain linear demand functions) lacks some robustness. Moreover, they interpreted somewhat differently the equation decomposing the various values of time: κ_i being the multiplier of the sole time technological constraint, its ratio with the income multiplier ($\frac{\kappa_i}{\lambda}$), which they name the *Subjective Value of Travel Time Saving*, is considered as the value of travel time (named ω_i in this chapter) and equalized with the sum of the *Value of Time as a Resource*, $\frac{\mu}{\lambda}$, and of the *Value of Time Assigned to Travel*, $-\frac{U_i'}{\lambda}$:

$$SVTTS = \frac{\kappa_i}{\lambda} = \frac{\mu}{\lambda} - \frac{\frac{\partial U}{\partial t_{tr}}}{\lambda} \tag{39}$$

Table 2.3:Average values of time for two income groups (Chile, 1991)

Income level	SVTTS	VTR	W	VTAW	VTAT
Medium	16.14	0.85	22.77	-21.92	-15.29
High	46.74	2.29	50.41	-48.12	-44.45

Note: Table 6 from Jara-Diaz and Guevara (2003). Value of one hour in 1991 Ch. dollar. SVTTS: Subjective Value of Travel Time Saving $\frac{\kappa_i}{\lambda}$; VTR: Value of Time as a Resource $\frac{\mu}{\lambda}$; W: wage rate; VTAW: Value of Time Assigned to Work $-\frac{U_{t_w}'}{\lambda}$; WTAT: Value of Time Assigned to Travel $-\frac{U_{t_{tr}}'}{\lambda}$.

The results in Table 2.3 indicate that the disutilities of work time is very important, such that it quite annulates the value of time as a resource $\frac{\mu}{\lambda}$. The disutility of travel time is also very large, (although a little smaller), so that

finally the total value of travel time, identified as κ_i, is entirely due to the intrinsic disutility of travel time. These results are not very interesting for the interpretation of the decomposition of time values.

Jara-Diaz's model could be applied to a statistics allowing the estimation of demand functions for goods and time expenditures, as soon as the individual wage rate, the market labor time and the minimum levels of goods and time uses are informed in the dataset. These various information could be obtained by an estimation (for instance of a Linear Expenditure System for the technological constraints). Finally, the same analysis could be carried over generalizing the Cobb-Douglas direct utility to a CES or translog specification in order to obtain non-linear demand functions of goods and time expenditures.

6 A generalization of Becker's model with a unique value of time

In order to analyze the endogeneity of the opportunity cost of time, I assume a Cobb-Douglas structure both for the utility and the domestic production functions of the final goods (which correspond to activities such as eating, transportation, housing, clothing or leisure activities). Note that all the parameters will be estimated locally (i.e. for each household in the dataset), so that this specification just assumes additive separability of the utility[21] and, for each household, the constancy, in a neighborhood of its equilibrium point, of the elasticities γ_i of the domestic productions in the utility, and the elasticities α_i and β_i of the two factors in the production functions. The optimization program is (all variables correspond to a household h which index is omitted in the equations):

$$\max_{m_i,t_i} u(Z) = \Pi_i a_i z_i^{\gamma_i} \; with \; z_i = b_i m_i^{\alpha_i} t_i^{\beta_i} \tag{40}$$

under the full income constraint:

$$\sum_i (m_i + \omega t_i) = w t_w + \omega(T - t_w) + y_0 \tag{41}$$

The full income y^f is the maximum monetary income which could be earned when working during all disposable time T (excluding sleeping time, although, as remarked by Aguiar and Hurst (2007a) sleeping time is in fact a choice variable over which individuals optimize. Sleeping time is generally excluded from domestic and leisure time in the time use literature), at the market wage rate net of taxes w with y_0 designing other monetary incomes:

$$y^f = wT + y_0 = y + \omega(T - t_w) = y + \omega \sum_i t_i \tag{42}$$

[21] Separability assumptions are highly disputable and generally empirically rejected (see our results based on the comparison between income and price elasticities in section 4.2). Nevertheless, it could be justified by the semi-aggregate nature of commodities, which diminishes the substitution between commodities compared to the substitution between similar goods within the same commodity.

It is assumed here that the agent's opportunity cost ω differs from her net wage, so that the full budget constraint writes:

$$\sum_i (p_i + \omega t_i) = y^f + (\omega - w)(T - t_w) = y^f + (\omega - w) \sum_i t_i \qquad (43)$$

In this formula, the full income is thus corrected by the difference between the market (w) and the personal valuation (ω) of the total domestic time $\sum_i t_i$: the agent subtracts the transaction cost between her leisure and market labor opportunity cost of time from her full income (this correction applies when the market labor supply t_w is predetermined, which defines the monetary income). This full budget constraint differs from the usual one whereby leisure or consumption time is valued by the agent's net market wage rate. This particularity allows a new method to be proposed to estimate the agent's opportunity cost of time.

Note that $T - t_w = \sum_i t_i = T_d$ and that both the market wage and the shadow wage (i.e. the opportunity cost of time ω[22]) appear in the budget equation: the shadow wage corresponds to the valuation of time in domestic production, and it is supposed to differ from the market wage w (see the discussion in the sub-next section)[23]. Note that prices p_i enter the utility function via $m_i = p_i x_i$ (instead of a specification of utility over quantities x_i) but the monetary prices are supposed to be the same for all households (since no direct information on prices per household is available in the dataset) so that this specification corresponds to a utility depending on quantities.
This specification supposes that market work is exogenous (it is in fact the residual after domestic times have been subtracted from disposable time, supposed to be given as total time less necessary time for sleeping). That corresponds to the assumption that households follow a two-stage budgeting approach where market working time does not affect monetary expenditures and domestic times[24] . This assumption is natural if the market time is fixed at an official amount as is the case in France (35 hours a week), but it can be discussed for all individual able to work supplementary hours or less than the official limit. It can indeed be assumed that while hours may be more regulated in full time jobs, part?time employment exists also in France which may be quite relevant in the decision between child care expenses and market work, for example. This is a limitation of the paper. Integrating the choice of market work would be possible but necessitates incorporating some variables which are not present in our dataset (but disposable in surveys on labor supply, which could be matched with our dataset).

[22]Supposed to be constant among all domestic activities. Child care is sometimes considered as more enjoyable than other domestic activities because of it is rationed. I thus assume that there exists no corner optimum in time allocation among domestic activities.

[23]Also, the household average opportunity cost of time may differ from the head's wage rate for complex family structures.

[24]However, as noted by a referee, Browning and Meghir (1991) demonstrated important non separability between labor supply and consumption choices that vary across goods.

6.1 Resolution of the model

In order to estimate the opportunity cost for time, the utility function is re-written:

$$u(Z) = \prod_i a_i z_i^{\gamma_i} = \prod_i a_i b_i^{\gamma_i} \left[\prod_i m_i^{\frac{\alpha_i\gamma_i}{\sum \alpha_i\gamma_i}}\right]^{\sum \alpha_i\gamma_i} \left[\prod_i t_i^{\frac{\beta_i\gamma_i}{\sum \beta_i\gamma_i}}\right]^{\sum \beta_i\gamma_i} = am^{'\sum \alpha_i\gamma_i} t^{'\sum \beta_i\gamma_i} \tag{44}$$

with $m^{'}$ and $t^{'}$ the geometric weighted means of the monetary and time inputs with weights $\frac{\alpha_i\gamma_i}{\sum \alpha_i\gamma_i}$ and $\frac{\beta_i\gamma_i}{\sum \beta_i\gamma_i}$. Deriving the utility over monetary income y and total leisure and domestic production timeT_d gives the opportunity cost of time:

$$\omega = \frac{\frac{\partial u}{\partial T_d}}{\frac{\partial u}{\partial y}} = \frac{\frac{\partial u}{\partial t'} \cdot \frac{\partial t'}{\partial T_d}}{\frac{\partial u}{\partial m'} \cdot \frac{\partial m'}{\partial y}} = \frac{m' \sum_i \beta_i\gamma_i}{t' \sum_i \alpha_i\gamma_i} \cdot \frac{\frac{\partial t'}{\partial T_d}}{\frac{\partial m'}{\partial y}} = \frac{\sum_i \beta_i\gamma_i}{\sum_i \alpha_i\gamma_i} \cdot \frac{yEl_{t'/T_d}}{T_d El_{m'/y}} \tag{45}$$

The ratio of the time and income elasticities: $\frac{El_{t'/T_d}}{El_{m'/y}}$ is estimated on our data as smaller but close to one. The value of time is thus proportional to the ratio of an index of monetary expenditures over time use[25] . Note that all parameters of the utility function are estimated locally by the first order conditions, so that the household's welfare depends both on the set of parameters (α, β, γ) and on its monetary and time expenditures m_i and t_i [26].

The Cobb-Douglas specification of the utility and the domestic production functions are necessary in order to be able to write the utility in terms of geometric means of expenditures and time uses. As these functions are estimated locally, they can correspond to any global specification for a static comparison between households (the Cobb-Douglas specification concerns only small deviations from the household's equilibrium). As noted by a referee to this paper, that Cobb-Douglas structure rules out complete specialization (in case where some commodity consumption is fully outsourced on the market), in the sense that the monetary or time input to each domestic production must be positive in order to insure a positive utility. But our commodities are defined broadly and each contains a large part of all consumptions, so that the expenditure and time inputs are always positive on our dataset (even on data containing both expenditures and time use, the inputs are positive for all households for such broad commodities: see for example an application of the model for such surveys from Burkina Faso in Gardes and Thiombiano, 2017).

[25]Note a similar proportionality in the first order condition for minimizing the cost of shopping intensity in Aguiar and Hurst model: using their equation 2 (2007b, page 1536), the opportunity cost of time can be written as the negative of the product of the elasticity of price with respect to shopping time cross the ratio of monetary over time expenditures.

[26]This direct utility can be used to estimate welfare calculations, which generalize usual welfare calculations based solely on wages and monetary expenditures (see the discussion by Aguiar and Hurst, 2007a and an application in Canelas et al., 2014).

6.2 Estimation of the opportunity cost of time

In order to calculate the parameters of the utility and domestic production functions, I consider the substitutions which are possible, first between time and monetary resources for the production of some activity, second between monetary expenditures (or equivalently time expenditures) concerning two different activities. First, the substitution between time and money in the domestic production function of activity i generates the first order conditions:

$$\frac{\frac{\partial u}{\partial t_i}}{\frac{\partial u}{\partial m_i}} = \omega \Rightarrow \frac{\alpha_i}{\beta_i} = \frac{m_i}{\omega t_i} \tag{46}$$

which implies:

$$\alpha_i = \frac{m_i}{m_i + \omega t_i},\ \beta_i = \frac{\omega t_i}{m_i + \omega t_i} \tag{47}$$

under the constraint of a constant economy of scale for each production function: $\alpha_i + \beta_i = 1$[27]. It is also assumed that all marginal productivities are positive: $\alpha_i,\ \beta_i,\ \gamma_i > 0$ and the utility is normalized with the constraint $\sum \gamma_i = 1$ (no economy of scale in the utility. Note that the estimation of these coefficients without that restriction gives a sum close to 1 up to 0.01, see Table A1 in Appendix).

Consider now the substitution between times t_i and t_j in the domestic production of two different final goods i and j: this substitution implies another condition between the parameters of the domestic production functions and the utility function:

$$\frac{\gamma_i}{\gamma_j} = \frac{\beta_j t_i}{\beta_i t_j} = \frac{\alpha_j m_i}{\alpha_i m_j} \tag{48}$$

so that:

$$\gamma_i = \gamma_1 \frac{m_i}{m_1} . \frac{\alpha_1}{\alpha_i} \tag{49}$$

for all $i \neq 1$.

All other substitutions between monetary and time resources devoted to different activities can be derived from (7) and (8).

In order to estimate these parameters, a possible method consists in the calibration of the opportunity cost of time in a first stage, for instance at the minimum wage rate which is constant over the population. Equations (6) thus give an estimate of α_i and β_i for each household, which gives γ_i by equation (7). In the second step, an estimate of the opportunity cost of time ω is given by equation (5) which allows the computation of the individual values of the parameters α, β,γ for each household using equations (6) and (8). These values enter equation (5) to give for each household the second step estimate of ω. The estimations on the French data as well as simulations[28] tend to show that this

[27] This restriction is necessary for the identification of the model, which could also be obtained fixing the sum of each couple (α_i, β_i) to some pre-determined value.

[28] Performed by J. Boelaert and P. Merrigan.

procedure may not converge rapidly to the true value of the opportunity cost of time.

Another method can be based on equations (6) and (8) which imply for all activities i, j:

$$m_i\gamma_j = m_j\gamma_i + \omega\gamma_i t_j - \omega\gamma_j t_i \tag{50}$$

This could be estimated as a system of $\frac{n(n-1)}{2}$ independent equations, calibrating γ_j at the average full budget share for one good[29] or as a system of $\frac{(n-1)(n-2)}{2}$ equations under the homogeneity constraint of the utility function: $\sum \gamma_i = 1$. In this system, the opportunity cost of time is over-identified, as well as all $\gamma_j, j > 1$[30]. We can also sum equations (9) over j with $\sum \gamma_j = 1$ to obtain $(n-1)$ independent equations (with no loss of information):

$$m_i = \gamma_i(m + \omega T) - \omega t_i,\ i = 1, ..., n \tag{51}$$

This system of equations can be estimated for the whole population, which gives a unique estimate of ω for all households[31] . The estimation without constraints (with different OCT ω_i for each commodity and no additivity constraint on coefficients γ_i) shows that the absence of economies of scale in the utility ($\sum \gamma_i = 1$) is verified while the differentiation of the OCT gives poor estimates (see Table A1). This is due to the fact that the true model with specific OCT per commodity does not correspond to a specification (10) with just different OCT for each equation (the estimation of this model on this dataset gives estimates of specific OCT all positive, except one not significant small negative value).

The estimation can also be performed on sub-populations or by a non-parametric local regression, which affords a set of estimates over the population. The resulting estimates of the opportunity cost of time ω and the parameters γ_j of the utility function are then used through equations (8) and (9) to calculate α, β and γ for each household. Finally, these estimates of parameters α, β and γ are used to estimate the opportunity cost of time $\omega(h)$ for each household h in the population by equation (5). A generalization of the model with several different opportunity costs of domestic activities is possible defining T and Y as time and monetary resources for a subset of activities and separating this subset from their complementary use for other activities in the utility. This gives rise to a system of equations which differ from the definition of the opportunity cost of time in equation (9). The test of these differentiated OCT is left for a future research.

[29]Note that taking together equations 7, 8 and 9 to calculate the value of ω gives rise to a highly non-linear equation in γ_i and ω which cannot be solved algebraically.

[30]Note that an estimation of this system of equation without constraint on the set of coefficients γ_i gives estimates the sum of which is close (up to 5%) to 1. On the contrary, estimating without the non-linear structure of the system of equations over the OCT and the γ, gives different estimates of the OCT, which shows that these constraints must be imposed in the estimation.

[31]The opportunity cost of time ω is over-identified in this system, which can be tested: a test for proportionality between equations shows that the structure of the system is not automatically verified in an unconstrained estimation (note that the large number of observations leaves no room for another result).

7 The case of multiple values of time: the CES specification

The CES specification of the domestic production function for commodity i is: $z_i = [a_i m_i^{\rho_i} + (1-a_i) t_i^{\rho_i}]^{\frac{1}{\rho_i}} = a_i^{\frac{1}{\rho_i}} \left[m_i^{\rho_i} + \frac{1-a_i}{a_i} t_i^{\rho_i} \right]^{\frac{1}{\rho_i}}$ which is embedded in a CES direct utility function: $U = \left[\sum_i A_i z_i^{\zeta_i} \right]^{\frac{1}{\Sigma \zeta_j}}$. The partial derivative of U over m_i is:

$$\frac{\partial U}{\partial m_i} = A_i \frac{\zeta_i}{\Sigma \zeta_j} z_i^{\zeta_i - \rho_i - 1} a_i \frac{U}{\Sigma A_j z_j^{\zeta_j}} m_i^{\rho_i - 1} \tag{52}$$

which gives, with a similar derivative over time, a specific value of the opportunity cost of time for each commodity i:

$$\omega_i = \frac{U'_{t_i}}{U'_{m_i}} = \frac{1-a_i}{a_i} \left(\frac{t_i}{m_i} \right)^{\rho_i - 1} \tag{53}$$

Note that a Cobb-Douglas specification of the utility function based on CES domestic production functions:

$$V = \Pi a_i^{\frac{1}{\rho_i}} \left[a_i m_i^{\rho_i} + (1-a_i) t_i^{\rho_i} \right]^{\frac{\gamma_i}{\rho_i}} \tag{54}$$

gives the same expression of the opportunity costs as the CES specification of the domestic production functions and the utility. The coefficients ρ_i of the production functions are linked to the elasticity of substitution σ_i by the relationship: $\rho_i = 1 - \frac{1}{\sigma i}$. These elasticities of substitution are estimated in Chapter III.

In order to calibrate a_i, I compute the ratio of time over monetary expenditures using the marginal productivities of time and money in the CES function as proxies for the cost of time and monetary price (valuing time by the household's wage rate W_i):

$$\frac{W_i t_i}{m_i} = \frac{W_i t_i}{p_i x_i} = \frac{\partial z_i}{\partial t_i} \frac{t_i}{x_i} = \frac{1-a_i}{a_i} (\frac{t_i}{m_i})^{\rho_i} \tag{55}$$

The ratio of a's thus writes:

$$\frac{1-a_i}{a_i} = \frac{W_i t_i}{m_i} (\frac{m_i}{t_i})^{\rho_i} \tag{56}$$

which corresponds to equation (11) if the wage rate is equalized with the opportunity cost of time. Note that a_i could also be calibrated for the average domestic time, monetary expenditures and market wage (or a fraction of it) in the population. a_i can also be calibrated at several values between 0 and 1, for instance 0.5, one third and two third, and the results compared.

Finally, the relationship between the set of opportunity costs of time ω_i for each commodity and a general cost ω can be recovered considering the full budget constraint:

$$y^f = \Sigma \left(m_i + \omega_i t_i \right) = W t_W + y_0 + \omega \left(T_d - t_W \right) \tag{57}$$

which implies:

$$\omega = \frac{1}{\Sigma t_i} \Sigma t_i \omega_i \tag{58}$$

The overall opportunity cost of time is thus a weighted average of partial costs, weighted by the time budget shares. This relation gives another solution for the computation of a_i: compute the individual opportunity costs of time ω_i using the CES specification with $a_i = 0.5$ and the resulting average overall $\overline{\omega}$ which can be compared to the ω estimated by equations (5) or (20). The ω_i are then corrected by the ratio of these two average ω (corresponding to the coefficient $\frac{a_i}{1-a_i}$ in the formula giving ω_i), in order to sum to the estimation given by the system of equations (9).

Note that a CES specification with domestic production functions depending on the monetary value of time use :

$$z_i = [a_i m_i^{\rho_i} + (1 - a_i)(\omega_i t_i)^{\rho_i}]^{\frac{1}{\rho_i}} \tag{59}$$

gives :

$$\omega_i = \frac{m_i}{t_i} \tag{60}$$

which corresponds to the Cobb-Douglas case with $\alpha_i = \beta_i = 0.5$ and to the CES specification with the elasticity of substitution $\sigma_i = -1$.

8 Discussion of the generalized model based on the Cobb-Douglas and CES specifications

The CES specification (multiple values of time) generalizes the Cobb-Douglas case (a unique value of time as a resource) supposing that the elasticities of substitution between time and money are constant across the population but different from one (that is the conclusion of chapter II where these elasticities are estimated for five countries). Note that the Cobb-Douglas direct utility implies linear demand functions. If the value of time is used as a price in the estimation of demand functions (either as a component of full prices or added as a supplementary price), one must consider that the model calibrating the value of time differs from the structural model from which the system of demand is derived. The household is thus supposed to estimate in a first step the value of time, and to derive its optimal consumption in a second step.

Identification of the value of time in the generalized model of section 2.4

The formula summing the three components of the value of time thus writes:

$$\omega_i = \frac{\mu}{\lambda} - \frac{\kappa_i}{\lambda} + \frac{U'_{2i}}{\lambda} \tag{61}$$

where ω_i is the valuation of time spent in activity i (which is the ratio of the marginal utilities of time and income), $\frac{\mu}{\lambda}$ is the *resource value of time*, corresponding to leisure activities (i.e. with no binding time technological constraint

and no intrinsic marginal utility of the activity), κ_i the Lagrange multiplier of the time technological constraint (and its ratio with the marginal utility of income λ the component of the value of time corresponding to this constraint and U_i' the intrinsic marginal utility of time use in activity i. The sum of these two last component can be named the *value of transferred time in activity i* (transferred from activity i to any activity for which the technological constraints do not bind) since it corresponds to the value of a unit decrease of the time minimum limit imposed to the time used in this activity or, according to DeSerpa's terminology, the *value of saving time from activity i*.

In order to interpret this formula, suppose a decrease of one unit of time $d\bar{t}_i = -1$ on the minimum time of a time technological constraint induces the same change of time allocated to activity i: $dt_i = -1$ since the constraint is binding. In that case, the change in the value of time $-\omega_i$ equals, first the loss of time as a resource: $-\frac{\mu}{\lambda}$, second the decrease of satisfaction corresponding to the intrinsic utility: $-\frac{U_i'}{\lambda}$, third the value issued from the relaxation of the technological constraint: $+\kappa_i$.

We can conclude that two types of the value of time must be estimated: first, the *resource value of time* ωwhich corresponds to the value of an increase of total disposable time, which can be consecrated to any activity, either for domestic production, pure leisure or market work. That value is measured by the ratio $\frac{\mu}{\lambda}$ by the estimation of the money and time Lagrange multipliers calculated on the total monetary expenditure and total disposable time for domestic production (supposing that market labor has been chosen in a first optimization step) in order to correspond to an unconstrained time budget (i.e. to nul multipliers κ_i of the technological time constraints attached to particular activities). The opportunity cost of time ω estimated my generalized model (with Cobb-Douglas specifications of the direct utility and the domestic production functions) corresponds to this definition[32]. On the other hand, the CES specification of this model makes possible the estimation of the values of time ω_i attached to each semi-aggregate activity, which may include both the marginal direct utilities U_i of the activity and the potential positive Lagrange multiplier κ_i of a time technological constraint (aggregated from individual multipliers of the items composing the semi-aggregate as explained in the following sub-section).

Aggregation of the Lagrange multipliers associated to the technological constraints

In case of an estimation on a set of semi-aggregate groups of activity (such as *Food* or *Transportation*), the time constraints are probably not binding since only some of the activities contained in the semi-aggregate are concerned by technological constraints. Name I_c the sub-group of activities concerned by a binding time constraint. The time use corresponding to the semi-aggregate activity I is greater then a minimum time since the non-binding sub-activities in I_{nc} add large amount of time to the aggregate. For instance, some ingredients such as sugar or water may be constrained by a minimum consumption, while other, such as meat or alcohol, are not. The multiplier κ_i of the techno-

[32]Note it may include the result of marginal (dis)utilities of domestic activities

logical constraints may be thus (at least for its linear approximation) equal to the weighted sum (weight being the frequencies of constraints activities) of the multipliers of disaggregated activities. The aggregate multiplier should be in this case be positive but smaller than those estimated on the constraint binding disaggregated activities.

The value of time is derived in the Cobb-Douglas model from the derivatives of the direct utility $U = U_1 + U_2$ over time which includes the average of the marginal *intrinsic* utilities: $\bar{U_2'}$ but not the Lagrange multiplier κ_i corresponding to the technological constraint on time use t_i. The average of the values of time ω_i over all activities may thus differ from the unique value of time as a resource $\omega = \frac{\mu}{\lambda}$:

$$\sum \frac{t_i}{T}\omega_i = \sum \frac{t_i}{T}\left[\frac{\mu + u'_{2i}}{\lambda} - \frac{\kappa_i}{\lambda}\right] = \omega - \frac{\bar{\kappa_i}}{\lambda} \tag{62}$$

so that the average of values of time ω_i is smaller than the value of time as a resource ω.

9 Interpretation of multiple opportunity costs of time by means of virtual prices

Suppose that the agent would prefer to consecrate a smaller duration $t*_i$ for activity i compared to the lower limit $\bar{t_i}$ imposed by the technological constraint $(t_i \geq \bar{t_i})$, the distance between the observed duration $t_i = \bar{t_i}$ and the optimal one $(dt_i = \bar{t_i} - t*_i > 0)$ corresponds, in case of a CES production function:

$$z_i = [ax_i^{\rho} + (1-a)t_i^{\rho}]^{-\frac{1}{\rho}} \tag{63}$$

to a change in the quantity of good x_i which maintains unchanged the domestic production z_i of activity i equal to:

$$dx_i = -\frac{1-a}{a}(\frac{x_i}{t_i})^{1-\rho}dt_i \tag{64}$$

and a change in the corresponding monetary expenditure $m_i = p_i x_i$:

$$dm_i = -p_i\frac{1-a}{a}(\frac{x_i}{t_i})^{1-\rho}dt_i = - - (\frac{p_i}{\omega})^{\rho}\frac{1-a}{a}(\frac{m_i}{\omega t_i})^{1-\rho}dt_i \tag{65}$$

This change in monetary expenditure can be divided by the change in time use in order to obtain the virtual price of time corresponding the marginal monetary cost of the technological constraint:

$$\pi_i^v = -\frac{dm_i}{dt_i} = (\frac{p_i}{\omega})^{\rho}\frac{1-a}{a}(\frac{m_i}{\omega t_i})^{1-\rho} \tag{66}$$

This virtual cost depends on the ratio of monetary over time expenditure and on the elasticity of substitution between the two factors: $\sigma_i = \frac{1}{1-\rho}$. The elasticity of the virtual price over the elasticity of substitution writes (with $|\sigma_i| = \frac{1}{1-\rho}$):

$$E_{\pi^v/|\sigma_i|} = (\frac{p_i}{\omega})^{\rho}\frac{1-a}{a}\log(\frac{m_i}{\omega t_i}) \tag{67}$$

which is positive (with $0 < a < 1$) if $m_i > \omega t_i$ and negative in the other case.

10 Empirical evidence

Value of time as a resource ω, France (2000)

An estimation is made for six activities: expenditures on health and education (where there is no actual expenditure in a number of cases) being included in the last group (Other expenditures). The first estimate is derived from the system of equations (11) which is estimated for 76 sub-populations defined by age groups, income per unit of consumption and family structure. The average estimate of ω is close to the minimum wage rate and smaller than the households' average wage rate.

Using this estimate to calibrate the opportunity cost in the definition of α_i and β_i (equations 9) and using the estimates of the utility coefficients γ_i by equations (11), gives, by equation (7), an average estimate of ω=5.0. This is close to the average estimate from equation (11): 6.03. These values correspond qualitatively to the answer of individuals in direct surveys on their substitution between time and money, which usually gives an opportunity cost of time that is much lower than the agent's wage rate, net of taxes.

The opportunity costs obtained on individual data (i.e. for each household) by this two-step method is positively indexed on the household's net wage (with an elasticity equal to 0.77) and also on the household's *relative* income (defined as the difference between a household's income and the average income of its sub-population). It also increases with family size, especially with respect to the number of children, which shows that home production is more valued in large families because of economies of scale i.e. the home production of public goods. This means that an increase of the family size may not help to produce more commodities with the same amount of input (which would correspond to economies of scale in domestic production), but that the public goods produced at home in large families profit more numerous families and thus increase the value of time devoted to this production. Table 2.5 contains the average estimates of the opportunity cost of time for various household types. It shows that, on average, it increases by two thirds between poor singles and rich large families, and more than doubles from the first decile to the last for the whole population.

In order to assess the variation of the estimated opportunity cost of time across the population, a non-parametric estimation of the same system (11) has been performed using the first step of the Lowess procedure[33]: for each observationk_0, a local weighting regression is performed where the weight for k_j is large if household k_j is close to k_0, small if not and zero outside a neighbouring of household k_0. The weighting is defined as an Epachnikov kernel depending on households' income per unit of consumption. This procedure gives rise to

[33]The Lowess procedure exists in the Stata programs but this procedure applies only to a simple linear regression model. The second step of the Lowess procedure eliminates outliers obtained in the first step regression, see Cleveland (1979).

local estimates of the opportunity cost of time for each household in the survey. The resulting estimates of ω averages 1.28 (s.e. 0.83) by equation (11) and 2.48 by the second step (equation 7). Both estimates are positively correlated with household's monetary income (elasticities 0.4 and 0.6 respectively) and with the age of the household head: the OCT increases until the age of 40, then decreases as found in previous estimations by cell. This is similar to the evolution found by Aguiar and Hurst (2007), as has already been remarked.

Table 2.4: Estimates of the value of time ω, France 2000

Method	Formula	ω	$\frac{\omega}{w}$	$\frac{\omega}{w^{min}}$	Min ω	Max ω
By cell	Equation (11)	6.03	0.63	0.87	1.82	11.33
s.e.		(2.14)				
Individual data	Equation (7)	5.00	0.52	0.73	0.02	483.53
s.e.		(6.05)				

Notes: by cell: equation (2.7) estimated on grouped data (76 cells defined by age groups, income per UC and 4 family structures: singles, couples without children, couples with children, more than 2 adults). *Individual data*: in equations (7) the opportunity cost is calibrated at the value obtained for data grouped by cell. Eleven negative values not reported. w=average household's wage rate; w^{min} =minimum wage rate in 2000 (4.97 euros, which amount to 72% of the average wage rate).

Table 2.5: Variation of the opportunity cost of time

population	Singles	Couples without child	Couples without 1 Child	Couple with 2 Children	Large families ***	Single poor ****	Other rich *****
6.24	5.98	6.24	6.98	6.47	5.48	5.19	8.53
(1.02)	(1.51)	(2.36)	(1.88)	(1.84)	(1.55)	(1.41)	(1.98)
4.74	4.84	4.47	4.92	4.93	5.09	4.74	4.93
(0.86)	(0.94)	(0.81)	(0.96)	(0.55)	(0.75)	(0.94)	(0.77)
6.92	6.99	6.81	7.08	7.20	6.60	6.04	10.50
(2.70)	(2.99)	(2.61)	(2.47)	(2.62)	(2.64)	(2.27)	(2.89)
15427	16580	17240	14463	12347	9126	4993	38029
50.9	55.4	57.6	41.8	39.9	41.4	49.8	53.0

* All population: 1st decile: 3.86; 9th decile: 8.77.

** Estimated *on individual data* by equation (7). All population: 1st decile: 3.89; 9th decile: 5.58.

*** Large families: couple with more than 4 children.

**** Singles with income per UC smaller than the first decile of the whole population.

***** Couples with income per UC larger than the last decile of the whole population.

Table 2.6: Estimates of coefficients α, β, γ of the home production model

	β_i	γ_i
Food	0.728	0.165
	(0.114)	(0.0008)
Housing	0.394	0.249
	(0.158)	(0.0011)
Transport	0.610	0.121
	(0.153)	(0.0007)
Clothing	0.588	0.051
	(0.287)	(0.0005)
Leisure	0.816	0.184
	(0.132)	(0.0013)
Other	0.598	0.229
	(0.254)	(0.0020)

Notes:

Empirical s.e. over the sample for coefficients β_i (which are calculated locally, for each household).

$\alpha_i = 1 - \beta_i$; γ_i is estimated by equation (11). $\sum \alpha_i \gamma_i = 0.440\,(s.e. = 0.110)$; $\sum \beta_i \gamma_i = 0.560\,(s.e. = 0.110)$

10.1 Estimate for five countries

Table 2.7: Value of time and market wage

Country	ω	$\frac{\omega}{wage}$	$\frac{\omega}{min\ wage}$	$1^{st} Centile \omega$	$99^{th} Centile \omega$
Burkina Faso	0.09	0.30	0.28	0.02	0.40
s.e.	0.01	-	-	-	-
Canada	10.93	0.69	1.58	1.45	59.82
s.e.	14.63	-	-	-	-
France	7.01	0.52	0.73	2.51	13.95
s.e.	8.67	-	-	-	-
Poland	4.26	0.43 -		0.34	38.42
s.e.	9.21	-	-	-	-
USA	12.75	0.74	2.88	2.80	31.17
s.e.	0.42	-	-	-	-

These estimates (Table 2.7) are all positive and approximately equal in average to one half of the average wage rate, as is also indicated by surveys on willingness to pay for an extra hour. The Cobb-Douglas model thus seems to provide a correct estimate of this value. This estimate is positively correlated

to the household's wage rate (elasticity equal to 0.65 for instance in France, see Gardes, 2019, pp. 106-107) and also to the relative position of the household in the income distribution (elasticity equal to 0.43). It increases with the family size (+0.37 euros per child in France), which may indicate economies of scale in domestic production. It increases till 45 years, then decreases.[34] The value of time also increases between countries relatively to the average wage rate, according to the degree of liberalization of their labor markets (see Gardes, 2018, new version 2022, section 3.2).

10.2 Estimates of multiple values of time by activity (France, 2000)

In order to estimate the opportunity costs ω_i by activity, formula (5) is used with a factor a supposed to be the same for all commodities and derived from the CES specification of domestic production functions:

$$\frac{1-a}{a} = \frac{\omega.t_d}{m}(\frac{m}{t_d})^{\rho} \tag{68}$$

with t_d total time use for domestic activities and m total household's expenditure.

These estimations (reported in Table 2.8 for France) are performed over the whole sample and for sub-populations defined by age classes, family composition and education level. The estimates are conformed to the previous estimate of ω using the Cobb-Douglas model: the first estimates by equation (21) of the individual ω_i are indeed similar on average to the Cobb-Douglas estimate of $\widehat{\omega}$, while the second by (27) averages two thirds. The two formulas give close estimates for each activity, except for Transport and Clothing. The individual opportunity costs of time vary a lot (from 2 to 12) between the different commodities, with a small opportunity cost for Food and Leisure and large ones for Housing and Transport. This means that the marginal utility of time changes significantly from one commodity to another[35].

That variation of the opportunity cost of time between different activities has been discussed by Becker (1965) in his original article: "Presumably, the cost of time varies considerably among commodities and at different periods: for example, the cost of time is often less on weekends and in the evening because many firms are closed then, which explains why a famous airline company/ocean liner intentionally includes a weekend in each voyage between the United States and Europe. The cost of time would also tend to be less for commodities that contribute to productive effort, traditionally called "productive consumption". A considerable amount of sleep, food and even "play" fall into this heading. The opportunity cost of time is less because these commodities indirectly contribute to earnings." Becker discussed this later (Becker, 1981), considering that the

[34] It could be expected that the value of domestic time is positively correlated to the probability to be present on the labor market, which is the case for both men and women in their middle age (presumably for women when children are not too young). This proximity to the labor market increases the possibilities to substitute market work to domestic labor.

[35] Conditional to the independence of the marginal utility of money as regards its destination to a particular expenditure.

degree of effort which is used up by an individual in a given activity may increase the value of the time used for that activity. Under this hypothesis and supposing that "married women spend less energy on each hour of market work than married man", he explains the difference between the men's and women's wage rates and the specializations of women in domestic productions. I propose in the following section 3.2 another explanation of the variability of the value of time between activities based on the relative possibility of substituting money and time in the domestic production of each activity.

Table 2.8: Estimates of the Opportunity Costs by activity for France (2000, euros)

Final good	Food	Housing	Transport	Clothing	Leisure	Other	All
Elasticity of substitution	-0.430	-0.488	-0.360	-0.759	-1.050	-0.491	-0.645
ω_A	2.48	9.90	4.05	6.47	0.75	2.41	5.08
ω_B	3.20	10.00	5.32	7.41	1.62	3.69	5.77
ω_A Education 1	2.10	6.90	3.43	3.81	0.43	2.15	3.78
ω_A Education 2	2.81	10.61	4.58	6.90	1.06	2.40	5.39
ω_A Education 3	3.13	17.65	4.76	12.48	1.32	2.79	8.24
ω_A Income quintile 1	1.61	5.21	2.43	2.42	0.24	1.03	2.79
ω_A Income quintile 2	2.24	7.42	3.27	4.38	0.47	1.88	3.91
ω_A Income quintile 3	2.73	10.71	4.27	7.11	0.81	2.64	5.45
ω_A Income quintile 4	3.65	19.24	6.34	14.55	1.75	5.07	9.60
ω_A Income quintile 5	3.43	16.34	6.05	12.04	1.43	4.15	8.16
ω_A Singles	2.12	14.27	2.11	4.64	0.52	1.83	7.04
ω_A Couple without children	1.88	6.67	3.39	3.50	0.43	2.63	3.65
ω_A Couple with children	3.20	11.73	5.42	9.91	1.26	2.52	6.11

Note: ω_A=estimation by formula (21) for the CES specification. The elasticities have been calibrated so that their average is equal to the global opportunity cost estimated for all commodities by the Cobb-Douglas specification (system of equation). ω_B=ratio of monetary and time estimation (formula 27) corresponding to the Cobb-Douglas specification (estimated by the system of equations 20) with $\alpha = \beta = 0.5$ and to the CES specification with an elasticity of monetary expenditures on time equal to 1.

Table 2.8 contains the estimates of the opportunity costs of time for the whole sample in France and different sub-populations grouped by income, family size and education. The average values of time increase between the first income quintile and the last one by 192%, with an income elasticity equal to 0.56. This variation can be explained by its positive correlation with the household's wage rate. This increase is particularly large for Housing, Clothing, Leisure and Other activities. They also increase with the education level, especially for Housing, Clothing and Leisure (and remain constant for transport and other activities). Finally, the presence of children seems to have an opposite effect on the opportunity cost, compared to the number of adults: indeed, it increases the opportunity cost for all activities, which can be related to the constraints on the time budget linked to the presence of children. On the contrary, couples without children have a smaller cost of time compared to bachelors, which indicates economies of scale in time, especially for activities such as Food and Housing. All activities characterized by a larger opportunity cost of time due to children (i.e. Food, Housing, transportation, clothing and leisure) have smaller opportunity cost of time for couples compared to singles, while the reverse correlation exists

for Other activities characterized by a decreasing opportunity cost of time with the number of children. This means that the increase of family size by means of the number of adults has an inverse impact on the value of time compared to the increased size by children. The presence of children thus seems to change a lot the domestic production technology.

Conclusion: critical view on the generalized model

The model presented in section 2.7 with a unique endogenous value of time (referred as the value of time as a resource in the literature) presents the advantage to be estimated locally, i.e. for each observation in the cross-sectional dataset. This allows evaluating the dependency of the opportunity cost of time with household's income, age or education, as well as its variation across socio-economic groups. The Cobb-Douglas version of the model can be nested into the more general CES specification which allows estimating values of time per activity and the various Lagrange multipliers which are attached to the resource or technological constraints. It is therefore possible to test the assumption of unicity of the value of time (that is the nullity of the sum of multipliers $(\mu_i + \kappa_i)$ or the elasticity of substitution between time and money expenditures equal to one) comparing the two models by usual tests on nested models. Apart from the assumption concerning the specification of domestic production functions and of the direct utility, the only assumption necessary to the estimation of all parameters is the absence of economies of scale in the domestic production ($\alpha_i + \beta_i = 1$ for all activities i).

Appendix: Bockstael et al. model

Boecktael et al. define a utility function:

$$u(x) = \frac{(\gamma_1 + \gamma_2)x_1 + \beta}{(\gamma_1 + \gamma_1)^2} \exp \frac{(\alpha + \gamma_1 x_2 + \gamma_2 x_3 - x_1 + \gamma_3 x_q + \varepsilon}{(\gamma_1 + \gamma_1)x_1 + \beta} \tag{69}$$

where x_1 is the consumption of the recreational good, x_2 and x_3 are bundles of goods with money price and no time cost (for instance insurance cost) or the reverse (for instance leisure time with no consumption of any market good), q an indicator of the quality of the recreational good (for instance the number of fishes caught during recreational fishing) and γ and β the parameters. ε is a normally distributed residual reflecting the distribution of preferences over the population.

Demand functions

The maximization of this utility yields the non- recreational demand functions for the two situations of the household on the labor market:

Corner solution: x_1 depending on separate money and time prices (p_1 and t_1):

$$x_1 = \alpha + \gamma_1 \bar{y} + \gamma_2 \bar{T} + \beta' \gamma_1 p_1 + \beta' \gamma_2 t_1 + \gamma_3 q + \varepsilon \tag{70}$$

Interior solution: x_1 depending on full cost incorporating money and time prices($p_1 + W_D t_1$) with W_D the wage received in discretionary employment:

$$x_1 = \alpha + \gamma_1(\bar{y} + W_D\bar{T}) + \beta^{'}\gamma_1(p_1 + W_D t_1) + \gamma_3 q + \varepsilon \tag{71}$$

Compensating variations

Substituting these demand functions into utility and inverting to obtain the money expenditure function allows to derive the compensating variation for the corner and interior solutions corresponding to the household's situation on the labor market[36]:

Corner solution: The compensating variation in money term with p_1^0 and x_1^0 the initial observed points and $\bar{p}_1$ the price which drives the household out of the market for x_1 writes:

$$CV_1^y = \exp\left[\gamma_1(\bar{p}_1 - p_1^0)\right](\frac{x_1^0 + \beta^{'}}{\gamma_1}) - \frac{\beta^{'}}{\gamma_1} \tag{72}$$

Interior solution: The compensating variation in money term writes:

$$CV_C^y = \exp\left[\gamma_1(\bar{p}_1 - p_1^0)\right](\frac{x_2^1 + \beta^{'}}{\gamma_1}) - \frac{\beta^{'}}{\gamma_1} \tag{73}$$

Corner solution: The compensating variation in terms of time writes:

$$CV_1^T = \exp\left[\gamma_1(\bar{p}_1 - p_1^0)\right](\frac{x_1^0 + \beta^{'}}{\gamma_1 W_D}) - \frac{\beta^{'}}{\gamma_1 W_D} \tag{74}$$

Interior solution: The compensating variation in terms of time writes:

$$CV_C^T = \exp\left[\gamma_1(\bar{p}_1 - p_1^0)\right](\frac{x_1^0 + \beta^{'}}{\gamma_2}) - \frac{\beta^{'}}{\gamma_2} \tag{75}$$

This method can be used once the following variables can be measured on the dataset: the quantity consumed or expenditure of some commodity (for instance total consumption, or leisure expenditures, or a set of consumption for the estimation of a system of demand under some constraints on the parameters supposed to be common to all expenditures), household's income, monetary prices and time uses. Note that the monetary prices are usually unknown on a cross-sectional survey. They could must be recovered, either by some information on regional prices, or by the aggregation of prices for the items of a semi-aggregate commodity using the household's specific budget shares of these items - both methods being disputable (see Crawford et al., 2003). (alternatively in Chapter VI, I use the full prices as proxies of commodity costs and derive the monetary price elasticities using Becker's definition of these full prices).

[36] A similar calculus can be made on equivalent variations.

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